Game Theory, Information, Incentives

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Course # 320.501: Analytical Methods (part 6)

The Moral Hazard Problem

- Moral hazard as a problem of hidden action (effort)
- Choice between two effort levels
- Effort as continuous variable
  - first-order approach
- Applications
  - incentives for managers
  - rationing in the credit market
  - introduction of know-how in technology transfer contracts

The Moral Hazard Problem

- Effort

  A's behavior not observable $\not\to$ contract

  labor contracts, insurance contracts

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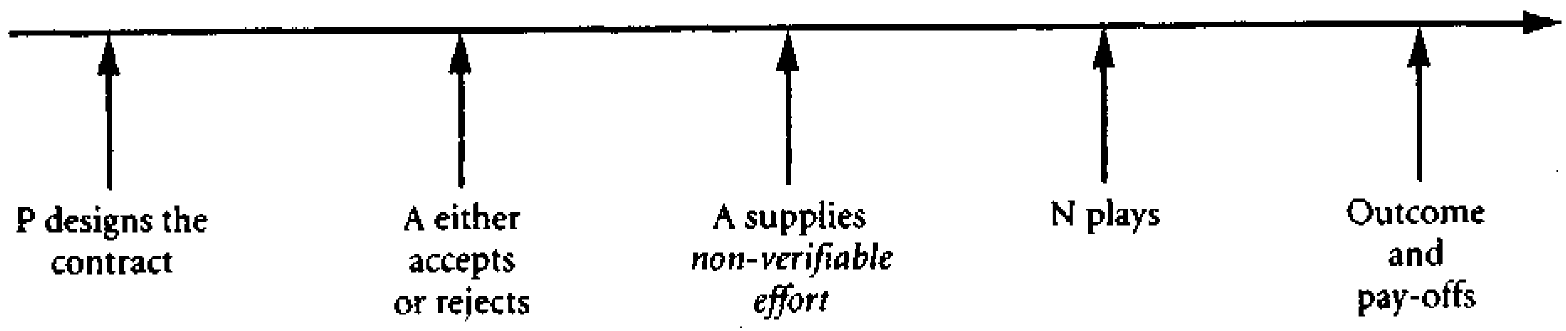
- Effort

  A's behavior not observable $\not\rightarrow$ contract

  labor contracts, insurance contracts

- Timing



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  - complete information (no hidden action): $w^o(x_i) = w^o(x_j)$
  - A chooses e^{MIN},
    as utility $= \sum_{i=1}^{n} p_i(e)\, u(w^o) - v(e) = u(w^o) - v(e)$

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    $\rightarrow$ inefficiency
  - trade off b/w efficiency and incentives

    (no efficient outcome, opt. contract in b/w fixed-wage & franchise)

Subgame perfect equilibrium

- 3^{rd} (final) stage: A chooses e

$$e \in \arg\max \left\{ \sum_{i=1}^{n} p_i(e')\, u(w(x_i)) - v(e') \right\}$$

  - incentive compatibility constraint

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- 2^{nd} stage: A accepts or rejects contract

$$\sum_{i=1}^{n} p_i(e)\, u(w(x_i)) - v(e) \geq \underline{U}$$

  - participation constraint

Subgame perfect equilibrium

- 1^{st} stage: P designs contract

$$\max_{e,\{w(x_i)\}_{i=1,\dots,n}} \sum_{i=1}^{n} p_i(e)\, B(x_i - w(x_i))$$

$$\text{s.t.} \sum_{i=1}^{n} p_i(e)\, u(w(x_i)) - v(e) \geq \underline{U}$$

$$e \in \arg\max \left\{ \sum_{i=1}^{n} p_i(e')\, u(w(x_i)) - v(e') \right\}$$

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  - $x_1 < x_2 < ... < x_n$
    $p_i^L \equiv p_i(e^L) > 0$, $p_i^H \equiv p_i(e^H) > 0$, $i = 1, 2, ..., n$

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    $p_i^L \equiv p_i(e^L) > 0$, $p_i^H \equiv p_i(e^H) > 0$, $i = 1, 2, ..., n$
  - first order stochastic dominance of p^H

$$\sum_{i=1}^{k} p_i^H < \sum_{i=1}^{k} p_i^L,\ k = 1, 2, ..., n-1$$

Digression: first order stochastic dominance

- $P^H(x)$, $P^L(x)$ cdf, $p^H(x)$, $p^L(x)$ pdf
  - $P^H(x)$ fosd $P^L(x)$ if $1 - P^H(x) > 1 - P^L(x)$ for each $x \in X$

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    $\Leftrightarrow \int p^H(x)dx < \int p^L(x)dx$ for all $x \in X$ (*)

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- discrete framework
  - condition (*) is: $\sum_{1=1}^{k} p_i^H < \sum_{1=1}^{k} p_i^L$ for $k = 1, ..., n-1$

  $\rightarrow$ for any $k < n$, probability that $x_i > x_k$ is higher under e^H than under e^L

Choice b/w two effort levels cont.'ed

- Suppose for P, $e^o = e^L$
  - incentive compatibility constraint satisfied
    $u(w^L) - v(e^L) > u(w^L) - v(e^H)$
    $\rightarrow$ symm. inf contract continues to be optimal
    (no incentive probl., no efficiency probl.)

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    $\sum_{i=1}^{n} p_i^H u(w(x_i)) - v(e^H) \geq \sum_{i=1}^{n} p_i^L u(w(x_i)) - v(e^L)$
  - $w^o = w^o(x_i)$, & needs to increase sufficiently

$$\mathcal{L}(\{w(x_i)\}, \lambda, \mu) = \sum_{i=1}^{n} p_i^H [x_i - w(x_i)] + \lambda \left[\sum_{i=1}^{n} p_i^H u(w(x_i)) - v(e^H) - \underline{U} \right]$$

$$+ \mu \left[\sum_{i=1}^{n} [p_i^H - p_i^L] u(w(x_i)) - [v(e^H) - v(e^L)] \right]$$

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- necessary foc

$$\frac{p_i^H}{u'(w(x_i))} = \lambda \, p_i^H + \mu \left(p_i^H - p_i^L \right), \quad i = 1, ..., n$$

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$$\frac{1}{u'(w(x_i))} = \lambda + \mu \left[1 - \frac{p_i^L}{p_i^H}\right], \quad i = 1, ..., n \quad (*)$$

$\rightarrow \mu > 0$

- $1/u'(w(x_i)) = \lambda + \mu \left[1 - p_i^L/p_i^H\right] \; (*)$
  - as $\mu > 0$: $w(x_i)$ changes in p_i^L/p_i^H

$\rightarrow$ fixed-wage contract not optimal, $w^o(x_i)$ depends on result

tradeoff b/w incentives and efficiency

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  tradeoff b/w incentives and efficiency
- Monotone likelihood ratio: p_i^L/p_i^H *decreases* in i
  - $x_i \uparrow \Rightarrow p_i^L/p_i^H \downarrow \Rightarrow$ RHS $(*) \uparrow \Rightarrow$ LHS $(*) \uparrow \Rightarrow u'(w(x_i)) \downarrow \Rightarrow w(x_i) \uparrow$

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$$u'(w(x_i)) = 1/\left[\lambda + \mu\left(1 - p_i^L/p_i^H\right)\right]$$
$$\Leftrightarrow w(x_i) = u'^{-1}\left[1/\left(\lambda + \mu\left(1 - p_i^L/p_i^H\right)\right)\right]$$

w^o depends on x_i to influence A's effort

Effort as continuous variable: first order approach

- $e \in [0, 1]$
  - incentive compatibility constraint

    $\sum_{i=1}^{n} p_i'(e)u(w(x_i)) - v'(e) = 0$

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- foc w.r.t. $w(x_i)$
$$\frac{1}{u'(w(x_i))} = \lambda + \mu \frac{p_i'(e)}{p_i(e)}$$

  $\mu > 0 \Leftrightarrow$ fosd;

  $p_i'(e)/p_i(e)$ rises in $i \Rightarrow w'(x_i) > 0$

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$$\sum_{i=1}^{n} p_i'(e)\,(x_i - w(x_i)) = -\mu \left[\sum_{i=1}^{n} p_i''(e)\, u(w(x_i)) - v''(e) \right]$$

  - increase in expected profit $=$ increase in expected cost (via incentive compatibility constraint)

Applications I: incentives for managers

- Symmetric information
  - P = shareholders, A = manager

    A: $U(w, e) = u(w) - v(e)$, usual properties

    P: $B(x, w) = p\,x - c\,x - w(x)$, sales x random var. with density $f(x; e)$

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$$\max_{w(x)} \int_X [p\,x - c\,x - w(x)] f(x;e)dx$$

$$\text{s.t.} \int_X u(w(x))\, f(x;e)dx - v(e) \geq \underline{U}$$

  - no incentive constraint (due to symmetric information)

Applications I: incentives for managers, cont.'ed

$$\mathcal{L} = \int_X [p\,x - c\,x - w(x)]\, f(x;e)dx + \lambda \left[\int_X u(w(x))\, f(x;e)dx - v(e) - \underline{U} \right]$$

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$$\frac{\partial \mathcal{L}}{\partial w} = -1 + \lambda u'(w) = 0 \quad \Rightarrow \quad \lambda = \frac{1}{u'(w)} > 0$$

from PC $w^o = u^{-1}(\underline{U} + v(e))\ \forall x_i$

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$\rightarrow$ fixed wage contract if e observable

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$$\frac{\partial \mathcal{L}}{\partial e} = 0 \quad \Rightarrow \quad \int_X [p\,x - c\,x] f_e'(x; e) dx = v'(e)/u'(w)$$

$\rightarrow$ expected marginal revenue of e = expected marginal cost of e

- Hidden action/effort

$$\max_{w(x)} \int_X [p\,x - c\,x - w(x)]\,f(x;e)dx$$

$$\text{s.t.} \int_X u(w(x))\,f(x;e)dx - v(e) \geq \underline{U}$$

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  - sufficient: $x = e + \epsilon$, ϵ normally distributed

Applications II: Rationing in the credit market

- Firm (A): choice b/w two risky investment projects, a, b, requiring I
  - payoff $\bar{X}_i = X_i$ with prob p_i $(0$ with prob $1 - p_i)$, $i = a, b$

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  - $U(R, i) = p_i \, (X_i - R)$

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  - Credit market: N firms, I investment per firm, L total amount of loans

    $I \leq L < N\, I$

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    credit rationing: firm cannot obtain all the money it wants
  - symmetric information (P observes choice of (a, b))

    $R = X_a$, $U(X_a, a) = 0$, no credit rationing
    (no firm makes positive profits with credits)

- Moral hazard: P cannot observe choice of A (a, b)

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- Subgame perfect equilibrium
  - step 3 (incentive compatibility)

    A chooses project $\in \{a, b\}$, depending on R

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} \text{ if } \begin{cases} p_a(X_a - R) \geq p_b(X_b - R) \\ p_a(X_a - R) < p_b(X_b - R) \end{cases} \Leftrightarrow \begin{matrix} R \leq \hat{R} \\ R > \hat{R} \end{matrix}, \quad \hat{R} \equiv \frac{p_a\, X_a - p_b\, X_b}{p_a - p_b}.$$

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  - step 3 (incentive compatibility)

    A chooses project $\in \{a, b\}$, depending on R

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} \text{ if } \begin{cases} p_a(X_a - R) \geq p_b(X_b - R) \\ p_a(X_a - R) < p_b(X_b - R) \end{cases} \Leftrightarrow \begin{matrix} R \leq \hat{R} \\ R > \hat{R} \end{matrix}, \qquad \hat{R} \equiv \frac{p_a\, X_a - p_b\, X_b}{p_a - p_b} .$$

  - step 2 (participation): $U(R, i) = p_i(X_i - R) \geq 0$

    if $R > \hat{R}$, $R \leq X_b$

- step 1 (contract design by principal)

  P chooses R so to maximize expected profit

$$\Pi^*(R) = \begin{cases} p_a\, R - I & \text{if } 0 \leq R \leq \hat{R} \\ p_b\, R - I & \text{if } \hat{R} < R \leq X_b \end{cases}$$

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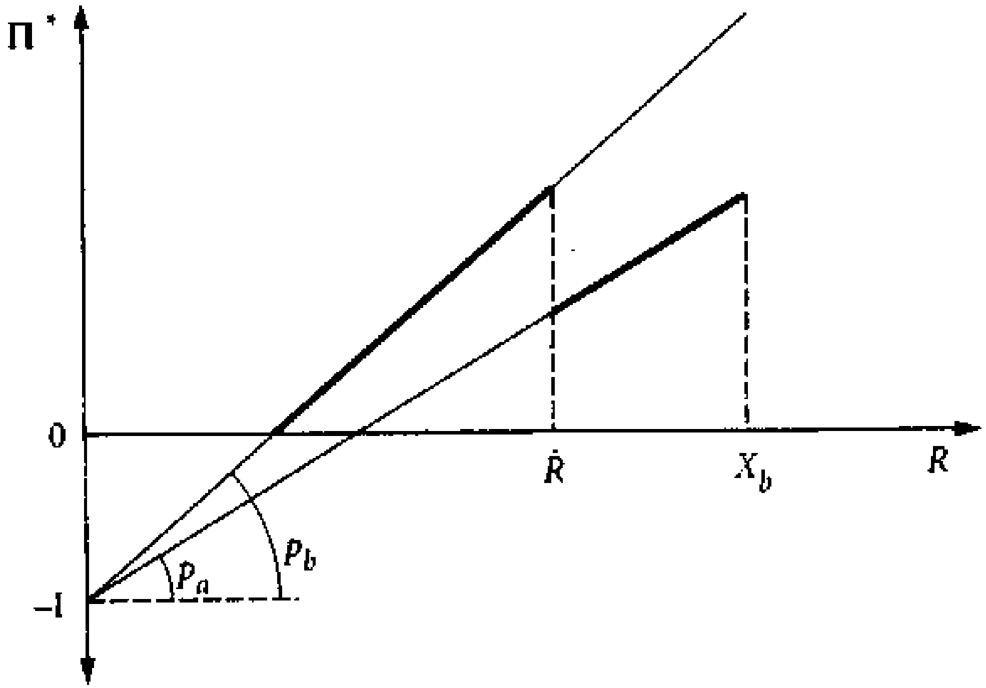

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  - Case 1: $R = X_b$

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    all firms ask for a loan, but $L < N\,I$
  - consequences of moral hazard
    - market interest rate changes $(R \gtrless X_a)$
    - banks may decide not to increase interest rate, even if firms would be willing to pay a higher interest rate

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- P's effort not observable

- Subgame perfect equilibrium
  - stage 4: A considers profit max Q

    $\hat{Q}(T, K, V) = \arg\max_Q \Pi(Q, T, K) = [a - C(T, K) - V^K]/2$

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    $$F^k + V^k\,\hat{Q}(T,k,V^k) - d(k) \geq F^k + V^k\,\hat{Q}(T,0,V^k) \Rightarrow K = k$$

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  - ... or $K = 0$

    $F^0 + V^0\,\hat{Q}(T,0,V^0) \geq F^0 + V^0\,\hat{Q}(T,k,V^0) - d(k) \Rightarrow K = 0$

    $\Leftrightarrow V^0 \leq 2d/[C(T,0) - C(T,k)]$

- stage 2: A's participation constraints

$$\Pi(Q, T, k) \geq \Pi(Q, 0, 0)$$

$$\Leftrightarrow F^k \leq [(a - C(T, k) - V^k)/2]^2 - [(a - C(0, 0))/2]^2$$

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- stage 1: P designs contract

  (F^k, V^k) or (F^0, V^0) depending on which yields higher payoff

- Case 1. (F^0, V^0) is optimal contract
  - consider $\hat{Q}$, participation constraint binds

$$\max_{V^0} [(a - C(T,0) - V^0)/2]^2 - [(a - C(0,0))/2]^2$$

$$+ V^0 [a - C(T,0) - V^0]/2 = -V^0/2 < 0$$

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  - optimal contract:

$$V^0 = 0\,, \quad F^0 = [(a - C(T,0) - V^0)/2]^2 - [(a - C(0,0))/2]^2$$

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  - if $K^o = 0$, moral hazard does not impose cost in terms of incentives

    P prefers not to distort A's MC!

- Case 2. (F^k, V^k) is optimal contract

$$V^k = \frac{2\,d}{C(T,0) - C(T,k)} > 0$$

$$F^k = \left[\frac{a - C(T,k) - V^k}{2}\right]^2 - \left[\frac{a - C(0,0)}{2}\right]^2 > 0$$

- Case 2. (F^k, V^k) is optimal contract

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  - only a royalty payment induces P to transmit K together w/ T (otherwise K not effectively transferred)

- Specifics of Application III
  - unverifiable behavior on P's part(!), not on A's part
  - no uncertainty in outcome

$\rightarrow$ tradeoff b/w incentives and efficiency does not imply distortion in optimal distribution of risk