Game Theory, Information, Incentives

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Dynamic games of complete information

- Games of complete and perfect information: backward induction
- Games of complete but imperfect information:
  - subgame perfection
- Repeated games
  - infinitely repeated games, Folk theorem

Games of complete and perfect information

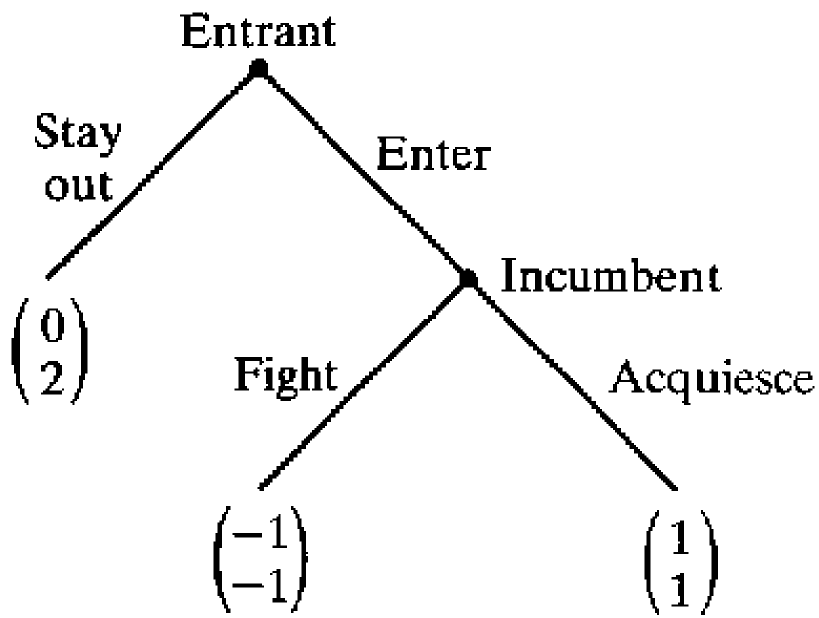
- Setup
  - moves occur in sequence

    previous moves are observed before the next move is chosen

    players' payoffs (types) are common knowledge
- Central theme: credibility
  - rule out non-credible threats
  - backward induction

Credibility

- Entrant-incumbent game



  - extensive form game
  - identify actions & strategies
  - identify both NE
  - identify non-credible threat

Backward induction

- Solution: backward induction rules out non-credible threats
  - Backward induction algorithm
    - **Definition.** $x =$ *penultimate node* if followed by endnode
    - $a_{i(x)}$ action at x, maximizing i's payoff with u_x payoff vector
    - replace x, actions and payoff vectors by $u_x \to$ reduced game with new x
    - repeat until action assigned to every node.
  - resulting set of actions: backward induction outcome

    associated joint strategy: backward induction strategy
  - if s is a backward induction strategy, s is a NE
  - if s is a NE $\not\Rightarrow$ s is a backward induction strategy
    - NE with non-credible threats don't survive backward induction

Example: Stackelberg duopoly

- Leadership in oligopolies (GM, US automobile industry)

  1. firm 1 chooses $q_1 \geq 0$
  2. firm 2 observes q_1, chooses $q_2 \geq 0$
  3. payoffs: $\pi_i(q_i, q_j) = q_i[P(Q) - c]$, where $P(Q) = a - Q$, $Q = q_1 + q_2$

- Backward induction

  firm 2 chooses π_2-max. q_2 for *every* q_1: that is, $q_2 = R_2(q_1)$

  firm 2's node is replaced by $R(q_1)$

  firm 1 chooses π_1-max. q_1 for $R(q_1)$

firm 2: $\max_{q_2 \geq 0} \pi_2(q_1, q_2) = \max_{q_2 \geq 0} q_2[a - q_1 - q_2 - c]$

$R_2(q_1) = (a - q_1 - c)/2$

firm 1: $\max_{q_1 \geq 0} \pi_2(q_1, R_2(q_1)) = \max_{q_1 \geq 0} q_1[a - q_1 - R_2(q_1) - c]$

- backward induction outcome:

$$\hat{q}_1 = \frac{a - c}{2}, \quad \hat{q}_2 = R_2(\hat{q}_1) = \frac{a - c}{4}$$

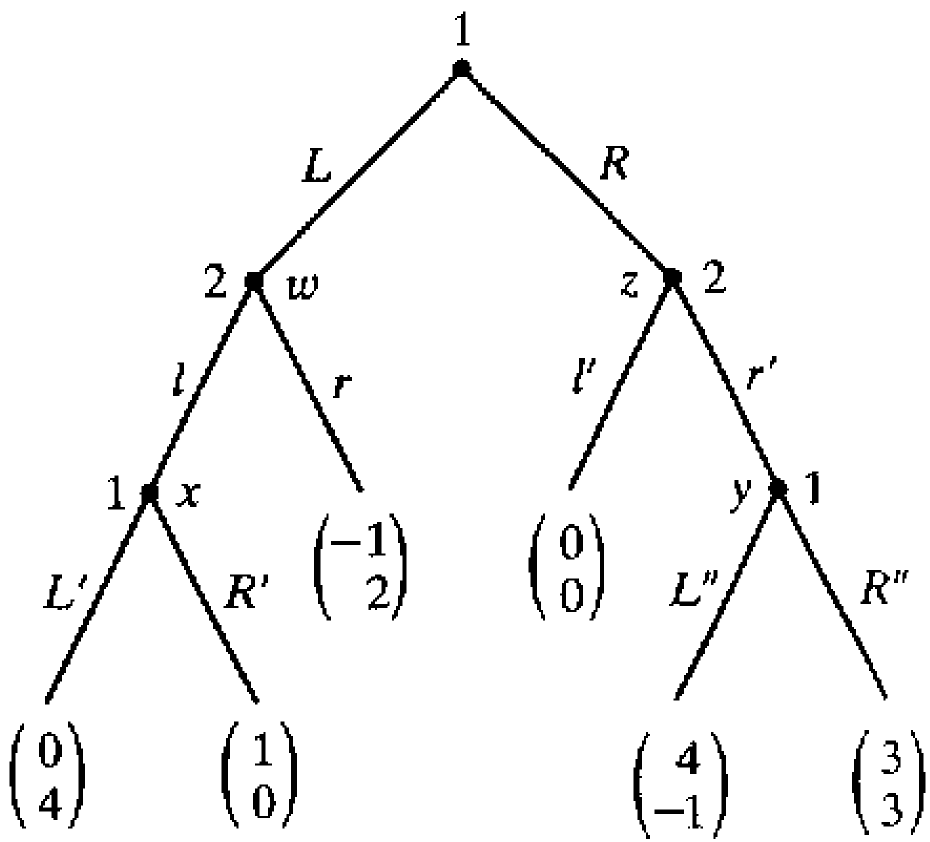
- backward induction strategy (NE):

$$\hat{q}_1 = \frac{a - c}{2}, \quad R_2(q_1) = \frac{a - q_1 - c}{2}$$

  - compare Stackelberg- with Cournot equilibrium

Example

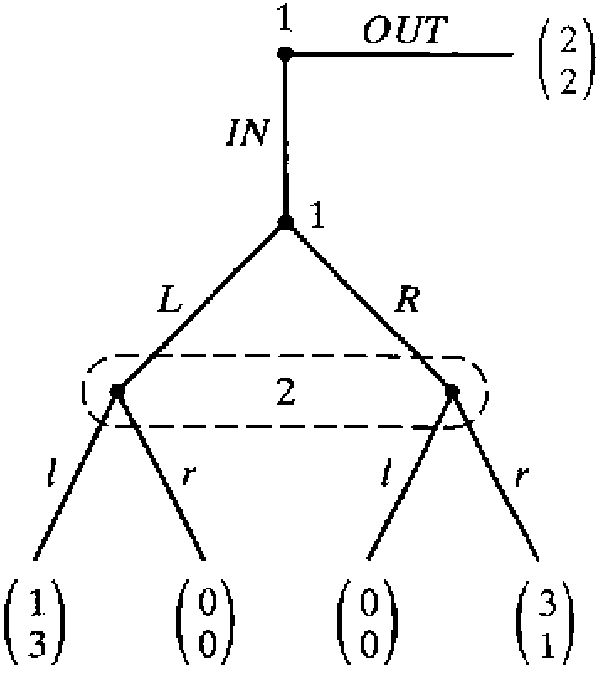
- Identify the backward induction outcome/strategy



Games with complete but imperfect information

- Imperfect information: previous move(s) not completely observed

  decision node not a singleton set → information set is not a singleton



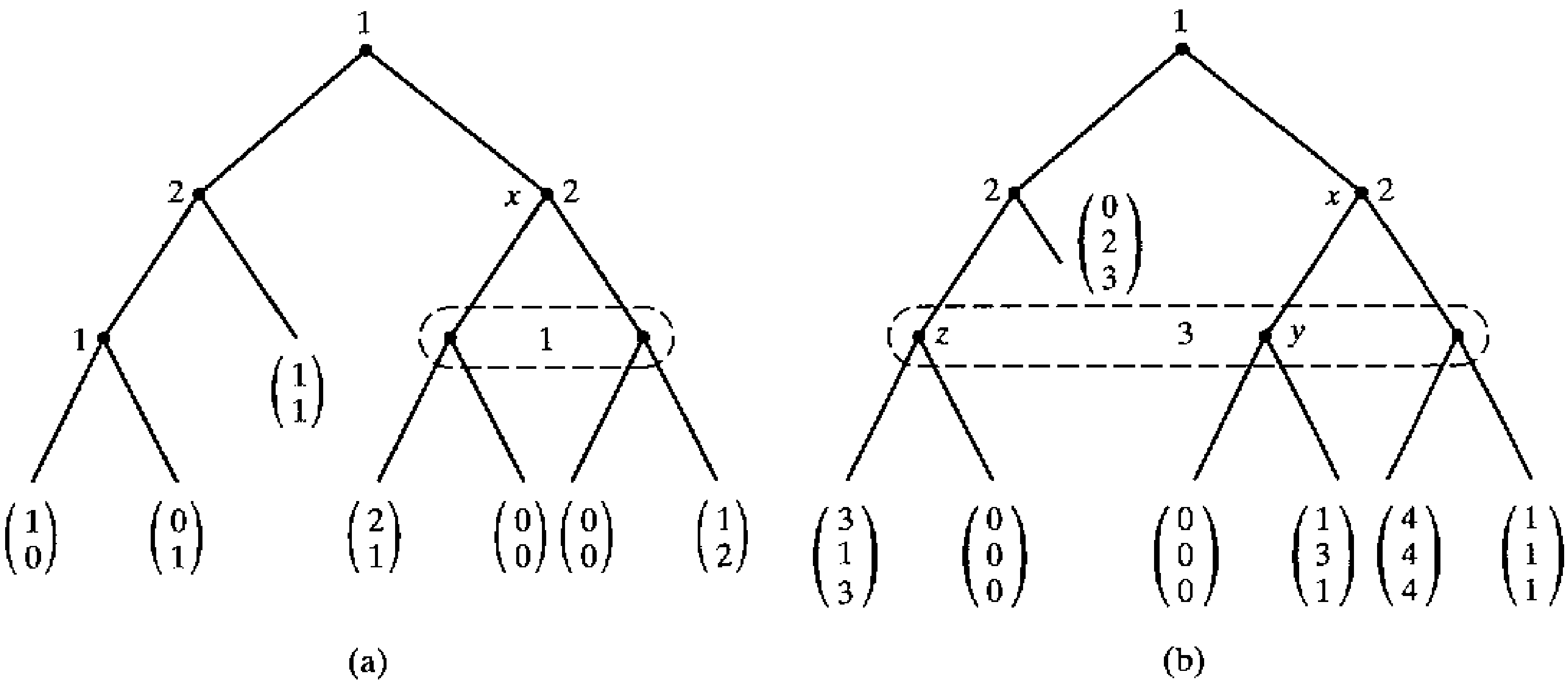
  ◦ backward induction – no penultimate node!

Subgames (Selten 1965,1975)

- Subgames
  - replace "penultimate node" by...
  - **Definition.** Node x defines subgame whenever
    (i) x belongs to singleton information set,
    (ii) if x' is a node following x, x' belongs to subgame,
    only followers belong to subgame,
    (iii) if node x'' belongs to same information set as x', x'' follows x.
  - game itself is considered a subgame

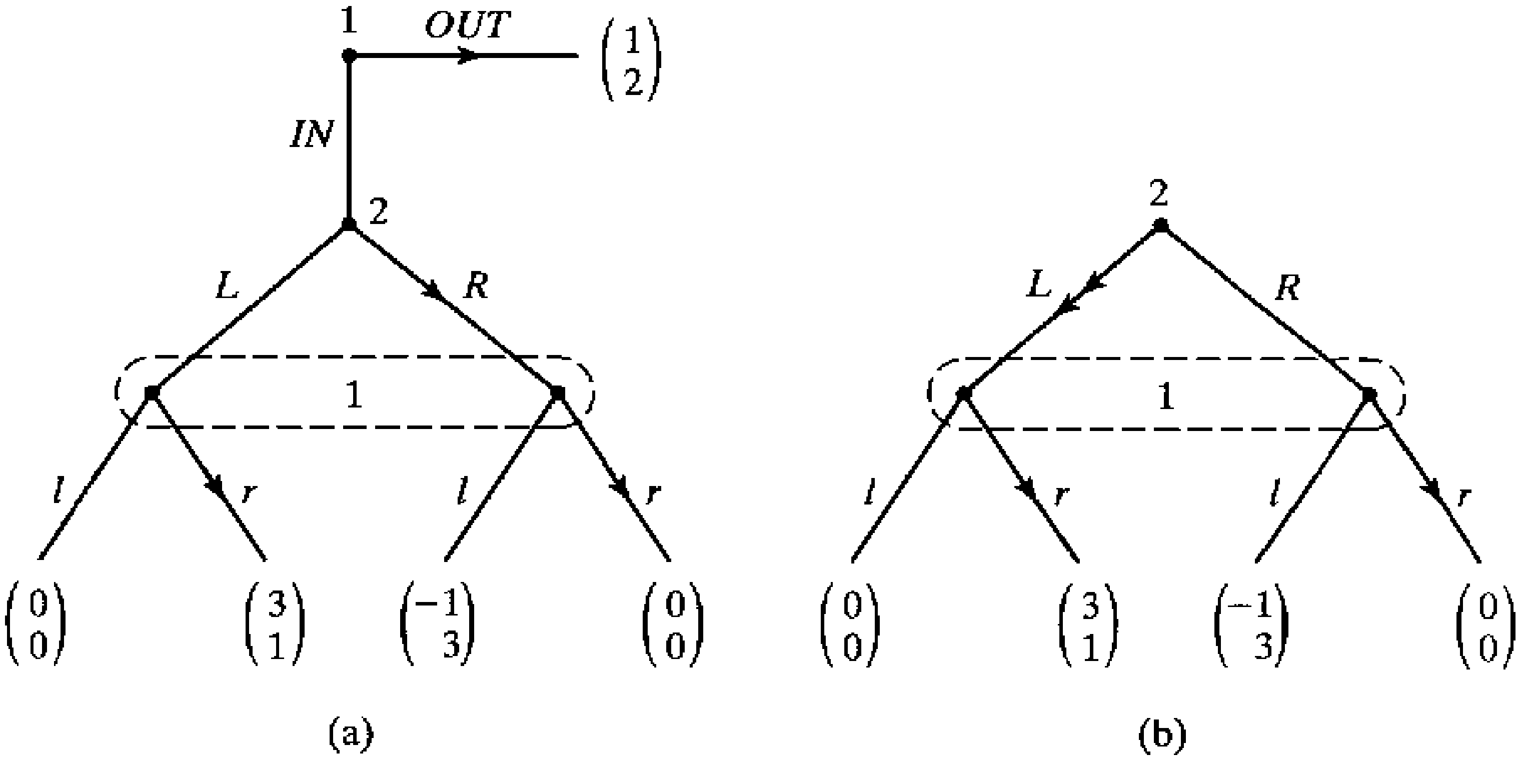
Example

- Identify all subgames



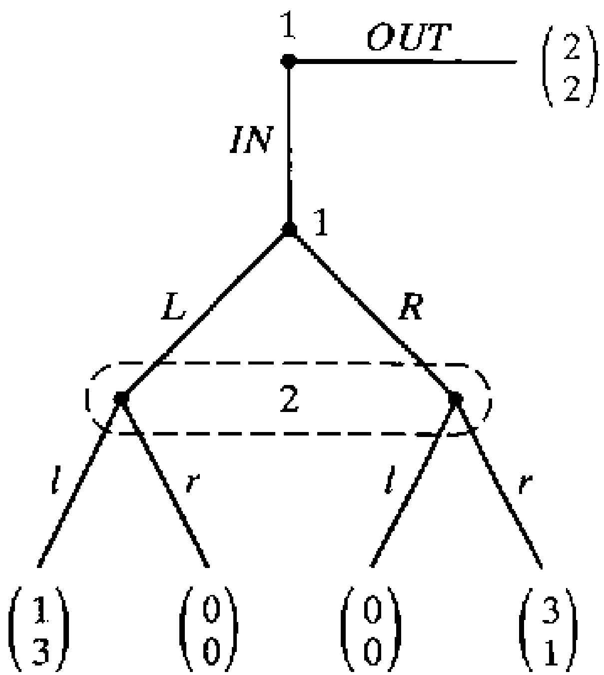
Subgame perfection

- **Definition.** A joint strategy s is a pure strategy subgame perfect equilibrium if s induces a NE in every subgame of the extensive form game.

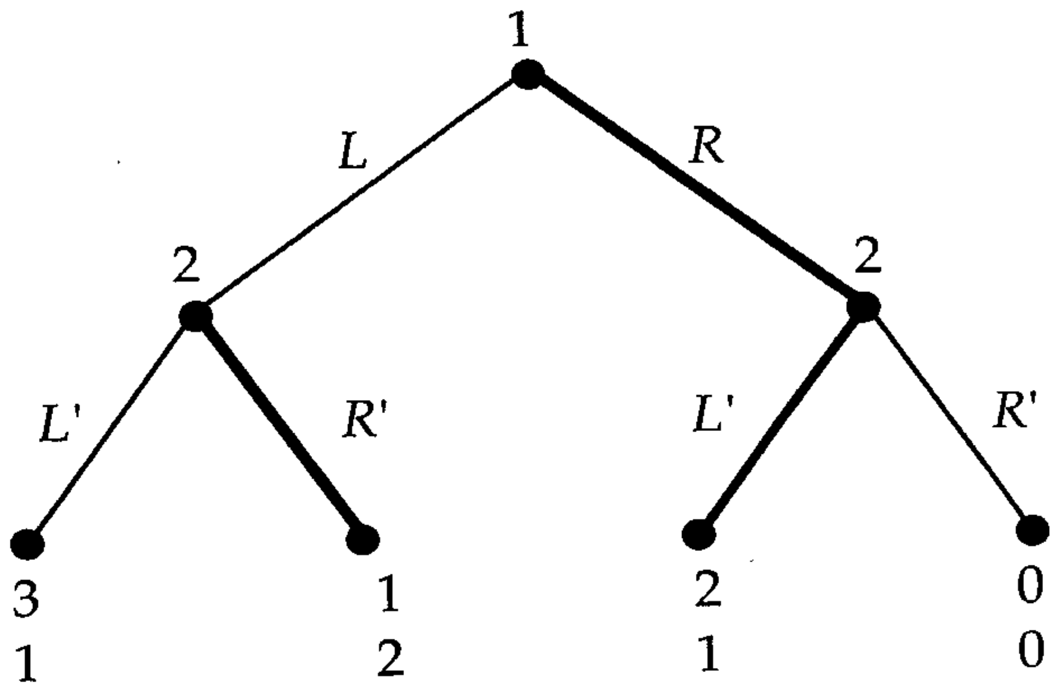


- $((OUT, r), R)$ is NE but not subgame perfect
  identify SPNE

- Identify NE and SPNE



- Identify NE and SPNE



  - identify the players' strategies
  - identify subgames
  - identify NE and SPNE

- Subgame perfection generalizes backward induction

Repeated games and Folk theorem

- Credible threats and promises influence future behavior
  - G stage game (to be repeated)
  - T # of stages, $G(T)$ repeated game
  - finitely vs. infinitely repeated games
- If G has unique NE, the finitely repeated game $G(T)$ has unique SP outcome: NE of G is played in every stage.

Player 1 \ Player 2	L2	R2
L1	1 , 1	5 , 0
R1	0 , 5	4 , 4

Player 1 \ Player 2	L2	R2
L1	2 , 2	6 , 1
R1	1 , 6	5 , 5

- Suppose G has unique NE. The infinitely repeated game $G(\infty, \delta)$ may have SP outcome that is not a NE of G.

  Intuition: cooperation vs. defection (trigger strategy)

		Player 2	
		L2	R2
Player 1	L1	1 , 1	5 , 0
	R1	0 , 5	4 , 4

"Infinitely" repeated games

- PV of infinite stream of payoffs, with δ discount factor

$$PV = \pi_1 + \delta\,\pi_2 + \delta^2\,\pi_3 + \ldots = \sum_{t=1}^{\infty} \delta^{t-1}\,\pi_t$$

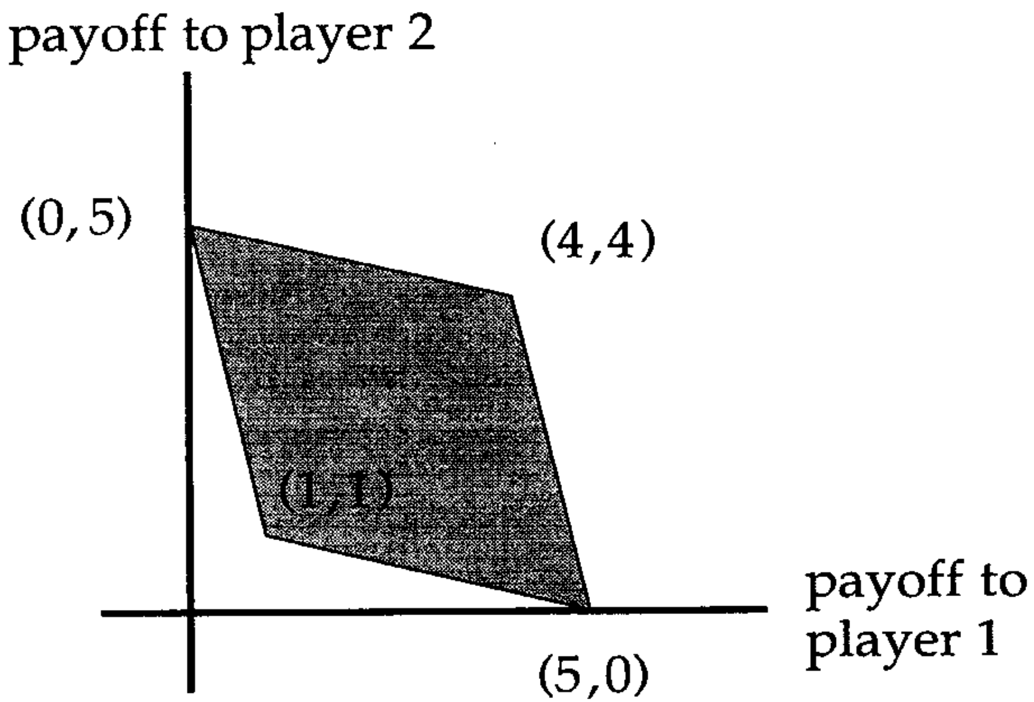
  - re-interpretation of $G(\infty)$ as $G(T)$
    - after each t, probability that game ends (continues) immediately is p (is $(1-p)$)
    - discount rate $= r$, then $\delta = (1-p)/(1+r)$

- Trigger strategies
  - roughly: cooperate as long as others cooperate, deviate forever once another player fails to cooperate
    - trigger strategy is a NE once δ close enough to 1
    - such a strategy is SP

- Trigger strategy: play R_i in first stage. In t^{th} stage, if outcome in all $t-1$ preceding stages was (R_1, R_2), play R_i; otherwise, play L_i.
  - if δ large enough, a one-time higher payoff from deviation does not compensate for an infinite sequence of lower payoffs as result from deviation $\rightarrow$ NE
  - every subgame of infinitely repeated game is identical to game as a whole
    - given NE, it's a NE of every subgame $\rightarrow$ NE is SPNE

Towards Friedman (1971) / Folk theorem

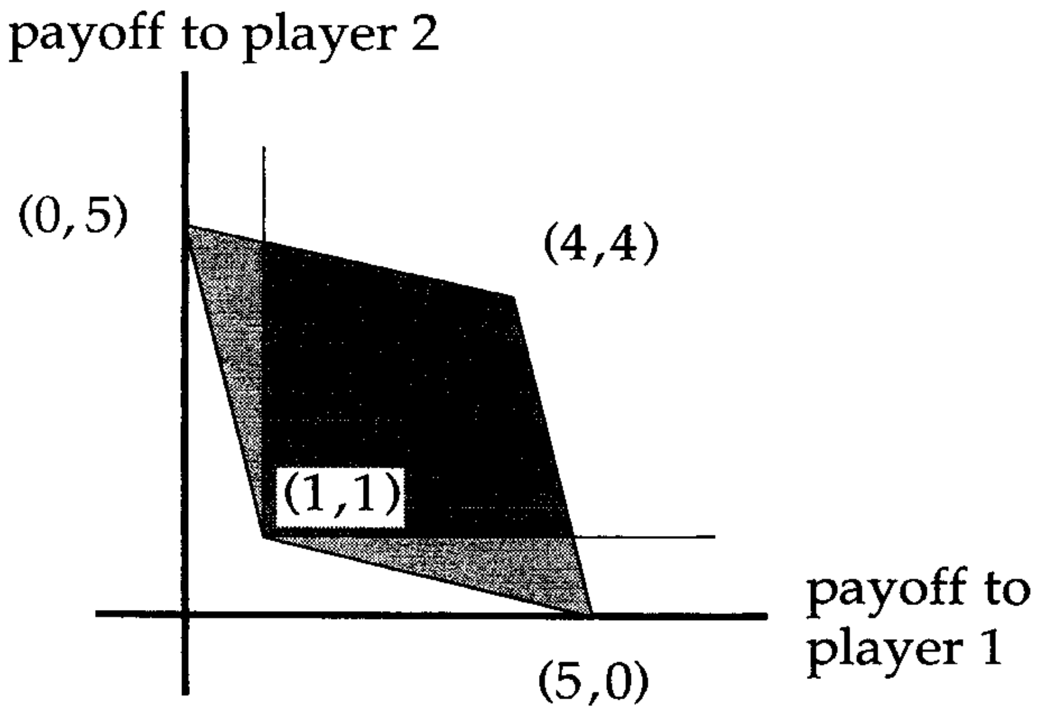
- *Feasible* payoffs in G, as convex combinations



Friedman (1971)

- Average payoff π
  - $\sum_{t=1}^{\infty} \delta^{t-1} \pi_t = \sum_{t=1}^{\infty} \delta^{t-1} \pi = \pi \sum_{t=1}^{\infty} \delta^{t-1} = \pi/(1-\delta)$
  - $\pi = (1-\delta) \sum_{t=1}^{\infty} \delta^{t-1} \pi_t = (1-\delta)PV$
- **Folk theorem.** Let G be finite stage game with complete information. Let $(e_1, ..., e_n)$ denote the payoffs from NE of G, and let $(x_1, ..., x_n)$ denote any other feasible payoffs. If $x_i > e_i$ for every player i, and if δ is sufficiently close to one, then there exists a SPNE of $G(\infty, \delta)$ that achieves $(x_1, ..., x_n)$ as the average payoff.

Infinitely repeated prisoner's dilemma



Example: Infinitely repeated prisoner's dilemma

Calculate δ for which $(4, 4)$ is SPNE

- PV of return on deviation $<$ PV of return from cooperation

  notice: $\sum_{t=0}^{\infty} \delta^t = 1/(1-\delta)$, and $\sum_{t=1}^{\infty} \delta^t = \delta/(1-\delta)$

- $5 + [\delta/(1-\delta)]\, 1 < [1/(1-\delta)]\, 4$

$\rightarrow$ $\delta > 0.25 \Leftrightarrow r <$ 300%

- Other examples: collusion b/w Cournot duopolists, time-consistent monetary policy