Game Theory, Information, Incentives

Ronald Wendner

Department of Economics
Graz University, Austria

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Signalling

- Signalling; education as signal
- Agents signal their characteristic
  - separating equilibria
  - pooling equilibria
  - intuitive criterion
- Application: prices as signals of quality

Signalling

- Adverse selection: asymmetric info yields for some agents
  - same expected utility as under symmetric info
    - several P, B-type of agent
  - higher expected utility
    - one P, G-type of agent (inf. rent)
  - lower expected utility
    - several P, G-type of agent
- Lower expected utility

  G-type interest to signal her characteristic
- Signal

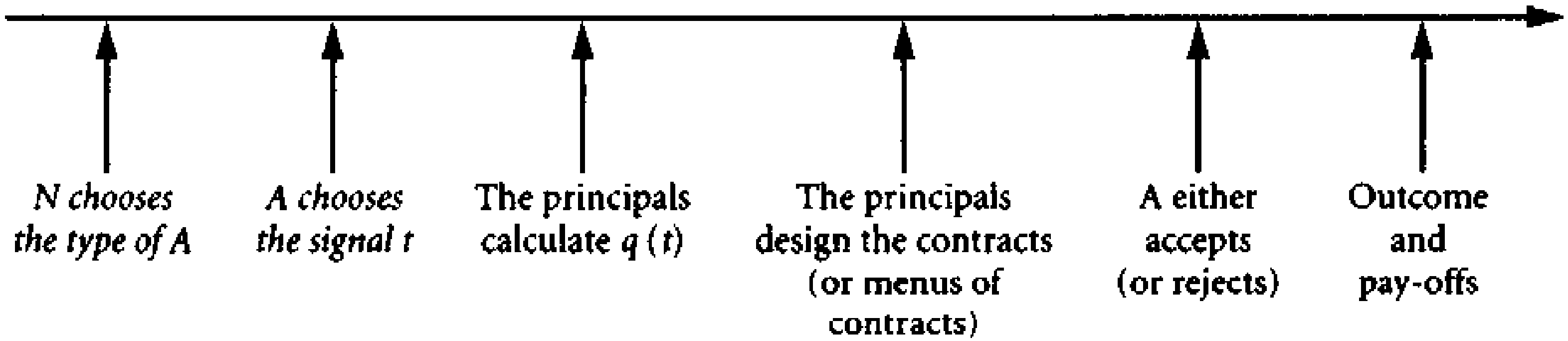
  costly observable activity/characteristic (prior to contract) that influences conditional belief of P about unobservable type of A

Preview example: education as a signal

- Setup
  - 2 types of workers (A): G w/ productivity=2, B w/ productivity=1 firm's (P) profit: $2 - w$ for G, $1 - w$ for B, (G, B) not observed by P
  - costly signal education: γ units cost: $\gamma/2$ to G, but γ to B education has no effect on result of relationship b/w P and A!
  - P beliefs: G if $\gamma \geq \gamma^*$ (offering $w = 2$), B if $\gamma < \gamma^*$ (offering $w = 1$)
  - G chooses $\gamma = \gamma^*$ if: $2 - \gamma^*/2 \geq 1 - 0$
  - B chooses $\gamma = 0$ if: $1 - 0 \geq 2 - \gamma^*$ $\Leftrightarrow 1 \leq \gamma^* \leq 2$
- Separating equilibrium (least cost-)
  - P belief: G if $\gamma = \gamma^* = 1$, B if $\gamma = 0$
  - P strategy: $w = 2$ if $\gamma = 1$; $w = 1$ if $\gamma = 0$
  - A $\gamma = \gamma^* = 1$ if G; $\gamma = 0$ if B
- Equilibrium concept: sequential equilibrium (PBE)

The signalling game

- Timing



- General setup: several P (zero profits)
  - result of relationship: F, S
  - A: G,B with $p^G > p^B$; P: Π_S, Π_F
- Symmetric information
  - contract menu $\{(w_S^G, w_F^G), (w_S^B, w_F^B)\}$
  - $w^{T^*} = w_S^T = w_F^T = p^T \Pi_S + (1 - p^T)\Pi_F, \; T \in \{G, B\}$
  - $U^{B^*} = u(w^{B^*}) < u(w^{G^*}) = U^{G^*}$

- Asymmetric information w/o signalling
  - contract menu $\{(w_S^G, w_F^G), (w_S^B, w_F^B)\}$
  - B receives U^{B^*} with $w_S^B = w_F^B = w^{B^*}$
  - G receives less: $U^G = p^G\, u(w_S^G) + (1 - p^G)\, u(w_F^G) < U^{G^*}$
- Signalling (separating equ.) $\Rightarrow$ symmetric information case

  B earns at least U^{B^*}

  G is willing to signal her type as long as cost $<$ utility gain
- Signal $t \in \{0, t'\}$; cost $v(0) = 0$, $v^G \equiv v^G(t') < v^B \equiv v^B(t')$
- Beliefs
  - prior q (share of G)
  - conditional $q(t)$ for all t, Bayes' rule
- Equilibria
  - separating $t^G \neq t^B$
  - pooling $t^G = t^B$

Separating equilibria

- Idea
  - belief: $q(t') = 1,\ q(0) = 0$
    strategy (P): $w(t') = w^{G^*},\ w(0) = w^{B^*}$
  - strategy (A): $t^G = t',\ t^B = 0$
  - sequential rationality:
    - $t^G \neq 0 \Leftarrow U^{G^*} - v^G \geq U^{B^*} \Leftrightarrow v^G \leq U^{G^*} - U^{B^*}$ (1)
    - $t^B \neq t' \Leftarrow U^{B^*} \geq U^{G^*} - v^B \Leftrightarrow v^B \geq U^{G^*} - U^{B^*}$ (2)
    - $v^G \leq v^B$ *and* v^G small enough, v^B large enough

- **Result.** If (1), (2) hold then there exists a sequential equilibrium:

$$(q(0), q(t'); w(0), w(t'), t^G, t^B) = (0, 1; w^{B^*}, w^{G^*}, t', 0).$$

- Proof sketch
  - for given beliefs, if t' observed: G; if 0 observed: B
    $=$ symmetric inf case; $(w(0), w(t')) = (w^{B^*}, w^{G^*})$ best response contract here and there
  - $(q(0), q(t')) = (0, 1)$ satisfy Bayes' rule as long as $t^G = t'$ and $t^B = 0$.
  - $t^G = t'$ and $t^B = 0$ is best response as (1), (2) hold

Pooling equilibria (signal, not contracts!)

- Signal not informative

  $\rightarrow$ adverse selection: (U^{B^*}, U^G)

  - $t^G = t^B = t'$ never pooling equ; B incentive to deviate

    $\rightarrow t^G = t^B = 0$

- Sequential (pooling) equilibrium:

  $(q(0), q(t'); w(0), w(t'), t^G, t^B) =$
  $(q, 0; \{(w_F^G, w_S^G), w^{B^*}\}, w^{B^*}, 0, 0).$

  - if $t = 0$, contract menu = AS $\{(w_F^G, w_S^G), w^{B^*}\}$
  - if $t = t'$, P believes B offers w^{B^*}

    $\rightarrow$ given belief, contract menu best response

- $t = 0$: prior $=$ posterior belief, Bayesian

  $t = t'$ not chosen $\Rightarrow$ every $q(t')$ is Bayesian
- given contracts, belief, $t = 0$ best response for both A

  $t^G = t'$ lower wage, $t^B = t'$ same wage; $+$ signalling cost

- Reasonable belief?
  - if no incentive for B but incentive for G to deviate then:

    belief $q(t') = 0$ not reasonable (intuitive)!

Intuitive criterion (Cho and Kreps 1987)

- B no incentive to signal t'
  - $v^B \geq U^{G^*} - U^{B^*}$ (2)

  G incentive to signal t' if
  - $U^{G^*} - v^G > U^G \Leftrightarrow v^G < U^{G^*} - U^G$ (3)
- only belief satisfying IC: $q(t') = 1$
- **Result.** If (2), (3) hold, there exists no pooling equilibrium satisfying the IC.

Separating and pooling equilibria, satisfying the IC

$$v^G \leq U^{G^*} - U^{B^*} \tag{1}$$

$$v^B \geq U^{G^*} - U^{B^*} \tag{2}$$

$$v^G < U^{G^*} - U^G \tag{3}$$

- Observe: (3) $\Rightarrow$ (1)
- **Result.**

  (a) If (2) and (3) hold, a separating but no pooling equ exists.

  (b) If (1) or (2) violated, a pooling but no separating equ exists.

  (c) If (1) and (2) but not (3): both pooling and separating equilibria exist.

  - (a) (3) $\Rightarrow$ (1) $\Rightarrow$ (i) (1), (2) hold: separating equ; (ii) G incentive to deviate: no pooling equ
  - (b) say (1) violated, then G no incentive to deviate, $t^G = t^B = 0$ is pooling (IC)
  - (c) cases in between

Application: Prices that signal quality

- Often, initial selling prices are low only to rise, once consumers know that quality is good.
- Setup
  - monopolist produces good: G or B quality, private information with cost $c^G > c^B$, price p
  - consumer prior belief: q that quality is G
    surplus for $(G, B) = (X - p, 0 - p)$, cons. buys if surplus ≥ 0
  - 2 periods, in each period P may buy 1 unit of commodity
    - if quality B in 1st period, no demand in second
    - if consumer does not buy in 1, she does not buy in period 2
    - $t = 2$: $p_2 = X$ for G, no market for B!
    - consumers pay in $t = 2$, firm's discount factor $\delta < 1$

Suppose $q\,X < c^B$

- No pooling equilibrium when $q\,X < c^B$
  - $p_i^G = p_i^B,\ i = 1, 2$
  - consumer's expected surplus $q\,X - p$, $\max p_1 = q\,X$
  - firm: $\Pi_1 = q\,X - c^B < 0$ does not sell at $t = 1$, $\Pi_2 = 0$
- Separating equilibrium: (G, B) signal via prices
  - if p^B observed, cons don't buy (neg surplus)

    no market for B
  - $\Pi^G = (p_1 - c^G) + \delta\,(X - c^G)$
    - participation: $\Pi^G \geq 0$
    - self-selection: $p_1 \leq c^B$ (every B demands a higher price than c^B)

    $\to$ $p_1 = c^B$

    $\to$ costly signal: as $p_1 < c^G$;

    only G prepared to signal

    given Π^G increases by enough

- $\Pi^G \geq 0 \Leftrightarrow \delta(X - c^G) \geq c^G - c^B$
  - separating equilibrium exists with
    - $p_1 = c^B$, $p_2 = X$
    - B does not enter market