

# An Introduction to *Mathematica* (complete)

(Quantitative Dynamic Macroeconomics, Lecture Notes, Thomas Steger, University of Leipzig)

This chapter provides a concise introduction to *Mathematica*. Instead of giving a rigorous discussion or a comprehensive summary of commands, the basic principles are demonstrated by going through some instructive examples. For a systematic introduction see, for instance, Gräbe and Kofler (2007) and Jankowski (1998) and the references cited therein. For a concise summary of notation and basic calculation see the supplement (Section 7 below).

## 1. First steps

### Basics 1

We set up the equation " $y=a+bx$ " and label it "equ"

```
equ = y == a + b x
```

```
y == a + b x
```

Next solve this equation for  $x$  by applying the command "Solve" and label the result "sol"

```
sol = Solve[equ, x]
```

```
{ { x ->  $\frac{-a + y}{b}$  } }
```

One can extract the RHS of this solution as follows

```
sol[[1, 1, 2]]
```

```
 $\frac{-a + y}{b}$ 
```

### Basics 2

Here we define a function  $f[x, y]$

```
f[x_, y_] := x0.3 y0.7
```

and then ask *Mathematica* for the value of  $f[x, y]$  at  $x=10$  and  $y=20$

```
f[10, 20]
```

```
16.245
```

Notice the two different assignments which have been used so far: "=" is an immediate assignment and ":=" denotes a delayed assignment. The difference becomes clear using an example

```
a = Random[]; b := Random[];
```

```
{a, a, a}, {b, b, b}
```

```
{{0.784854, 0.784854, 0.784854}, {0.795794, 0.953583, 0.379506}}
```

## 2. Equation solving

### Single equations

#### ■ Analytical solution

At first, label the LHS of the quadratic equation " $x^2 + ax + b=0$ " to read "exp"

```
Clear[exp, a, b, x]; exp = x2 + a x + b;
```

Notice the assignment " $\text{exp}=x^2 + ax + b$ "! To check this assignment, evaluate the expression "exp"

```
exp
```

```
b + a x + x2
```

Here we use the command "Solve" to solve the equation " $\text{exp}=0$ " w.r.t. x

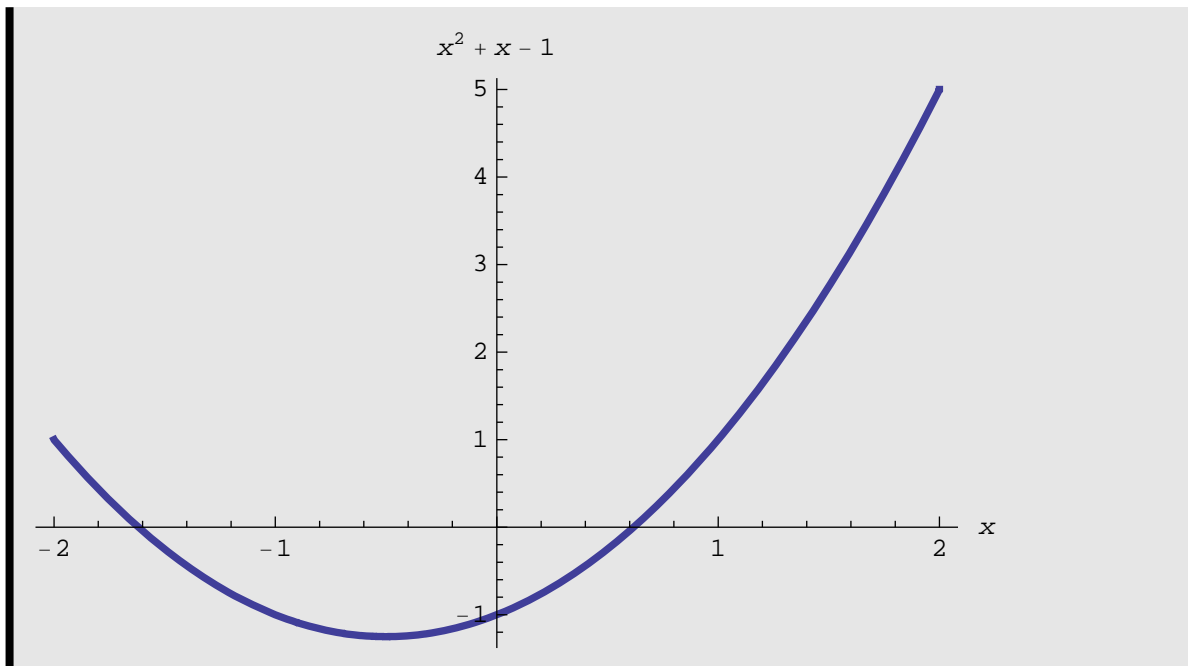
```
Solve[exp == 0, x]
```

```
{{x →  $\frac{1}{2} \left( -a - \sqrt{a^2 - 4b} \right)$ }, {x →  $\frac{1}{2} \left( -a + \sqrt{a^2 - 4b} \right)$ }}
```

We visualize the solution by plotting the quadratic equation. To plot the expression  $x^2 + ax + b$  over  $x$ , we must specify the parameters  $a$  and  $b$  numerically

```
Clear[a, b]; a = 1; b = -1;
```

```
Plot[exp, {x, -2, 2}, AxesLabel -> {x, exp}, PlotStyle -> Thickness[0.008]]
```



### ■ Numerical solution

One can also determine the solution to the quadratic equation numerically. To this end, we apply the command "FindRoot".

```
Clear[a, b]; a = 1; b = -1; FindRoot[exp == 0, {x, -2}]
```

```
{x -> -1.61803}
```

Observe that FindRoot does only find one solution! If we use another starting value for FindRoot, it may detect the other solution.

```
Clear[a, b]; a = 1; b = -1; FindRoot[exp == 0, {x, 2}]
```

```
{x -> 0.618034}
```

## System of equations

### ■ Analytical solution

Consider the following non-linear system  $\alpha_1 x_1^{\alpha_1-1} x_2^{\alpha_2} = a$  and  $\alpha_2 x_1^{\alpha_1} x_2^{\alpha_2-1} = b$ . Notice that this system represents the first-order conditions of the problem  $\max_{(x_1, x_2)} \{x_1^{\alpha_1} x_2^{\alpha_2} - a x_1 - b x_2\}$ .

```
Clear[a, b]; sys = {α1 x1α1-1 x2α2 == a, α2 x1α1 x2α2-1 == b};
```

The system is solved by applying "Solve"

```
sol = Solve[sys, {x1, x2}] // FullSimplify
```

$$\left\{ \left\{ \begin{array}{l} x1 \rightarrow e^{\frac{-(-1+\alpha_2) \text{Log}[a] + \alpha_2 \text{Log}[b] + (-1+\alpha_2) \text{Log}[\alpha_1] - \alpha_2 \text{Log}[\alpha_2]}{-1+\alpha_1+\alpha_2}}, \\ x2 \rightarrow e^{\frac{\text{Log}[b] - \text{Log}[\alpha_2] + \alpha_1 (\text{Log}[a] - \text{Log}[b] - \text{Log}[\alpha_1] + \text{Log}[\alpha_2])}{-1+\alpha_1+\alpha_2}} \end{array} \right\} \right\}$$

*Mathematica* gives the solution in a somewhat unusual form. A manual derivation (see supplement below) and numerical evaluation indicates that the result is correct.

Here we evaluate the solution for a baseline set of parameters

```
{α1, α2, a, b} = {0.3, 0.4, 0.1, 0.3};
```

```
{sol[[1, 1, 2]], sol[[1, 2, 2]]}
```

```
{13.2077, 5.87009}
```

Manual solution (see below) gives

$$\left\{ \left( \frac{b}{\alpha_2} \right)^{\frac{-\alpha_2}{-\alpha_1 - \alpha_2 + 1}} \left( \frac{a}{\alpha_1} \right)^{\frac{(-\alpha_1 - \alpha_2 + 1) + \alpha_1 \alpha_2}{(-\alpha_1 - \alpha_2 + 1)(\alpha_1 - 1)}}, \left( \frac{b}{\alpha_2} \right)^{\frac{\alpha_1 - 1}{-\alpha_1 - \alpha_2 + 1}} \left( \frac{a}{\alpha_1} \right)^{\frac{-\alpha_1}{-\alpha_1 - \alpha_2 + 1}} \right\}$$

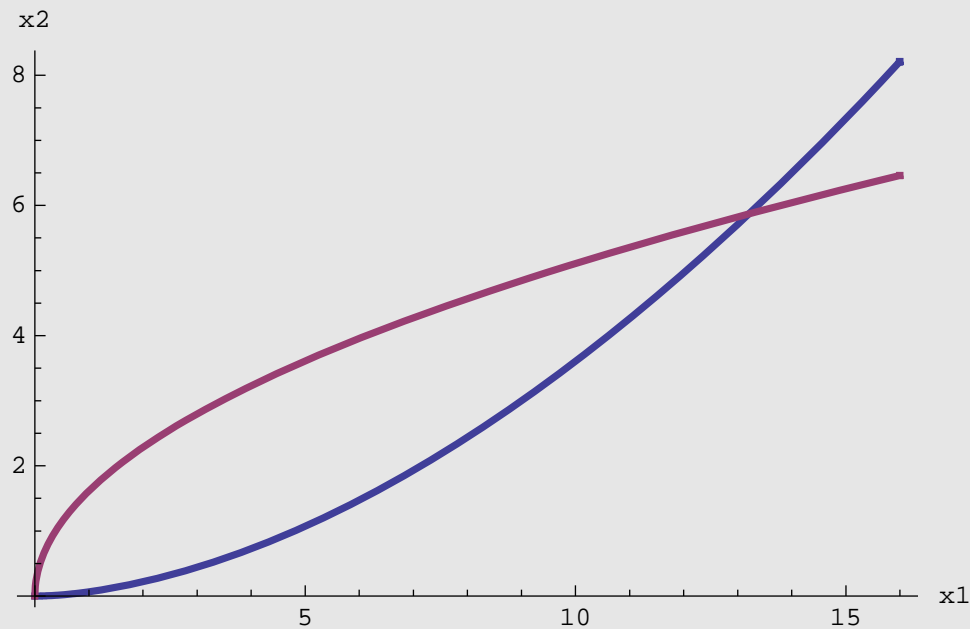
```
{13.2077, 5.87009}
```

Graphically, the solution is the intersection of the curves, given by the two first-order conditions, in the  $(x_1, x_2)$ -plane. To illustrate we plot the two curves

```
curve1 = Solve[α1 x1α1-1 x2α2 == a, x2] [[1, 1, 2]];
curve2 = Solve[α2 x1α1 x2α2-1 == b, x2] [[1, 1, 2]];

```

```
Clear[α1, α2, a, b]; {α1, α2, a, b} = {0.3, 0.4, 0.1, 0.3};  
Plot[{curve1, curve2}, {x1, 0, 16}, AxesLabel → {x1, x2},  
PlotStyle → Thickness[0.008]]
```



- Supplement: manual derivations (available upon request)
- Simulation: sequence of numerical solutions (available upon request)

---

### 3. Differentiation and integration

#### A simple example

Differentiate the expression " $x^2 + a x + b$ " with respect to  $x$  and name the result "diff"

```
Clear[a, b]; diff = D[x2 + a x + b, x]
```

```
a + 2 x
```

Next, form the (indefinite) integral of the expression "diff" with respect to  $x$

```
Integrate[diff, x]
```

```
a x + x2
```

There are several distinct ways to form a partial derivative

```
{D[Sin[x], x], ∂x Sin[x], Sin'[x]}
```

```
{Cos[x], Cos[x], Cos[x]}
```

The total derivative can be formed using the command "Dt"

```
Dt[f[x, y, z]]
```

```
Dt[z] f(0,0,1)[x, y, z] + Dt[y] f(0,1,0)[x, y, z] + Dt[x] f(1,0,0)[x, y, z]
```

Note the notation: Dt[z] means dz and f<sup>(0,0,1)</sup>[x, y, z] signifies  $\frac{\partial f(\cdot)}{\partial z}$ .

## Some Cobb-Douglas algebra

At first, we define Y to denote output according to a standard Cobb-Douglas production function

```
Clear[Y, A, L, K, α]; Y = A Lα K1-α;
```

The competitive wage and rental price of capital then is

```
{w = D[Y, L], r = D[Y, K]}
```

```
{A K1-α L-1+α α, A K-α Lα (1 - α)}
```

The production elasticity of labor and capital are defined as  $\eta_{YL} := \frac{dY/Y}{dL/L} = \frac{dY/dL}{Y/L}$  and  $\eta_{YK} := \frac{dY/Y}{dK/K} = \frac{dY/dK}{Y/K}$

```
{ $\frac{D[Y, L]}{Y/L}, \frac{D[Y, K]}{Y/K}$ }
```

```
{α, 1 - α}
```

Euler's theorem

```
w L + r K // Simplify
```

```
A K1-α Lα
```

A CES example

```
Clear[Y, w, r]; Y = (a Lα + (1 - a) Kα)1/α;
```

```
{w = D[Y, L], r = D[Y, K]}
```

```
{a L-1+α ((1 - a) Kα + a Lα)-1+1/α, (1 - a) K-1+α ((1 - a) Kα + a Lα)-1+1/α}
```

```
w L + r K == Y // Simplify
```

```
True
```

Euler's theorem holds, of course, also true in the general CES case.

---

## 4. Lists, functions, and some functional programming

### Lists

A list is a collection of objects. In *Mathematica*, a list is the fundamental data structure used to group objects together. As a first and simple example consider

```
a = {2, 4, 6, 8, 10}; b = {{1, 2}, {0, 4}};
```

An extensive set of built-in functions is available to form and manipulate lists. For instance, to access single elements of a list we can use "Part" command

```
Part[a, 1]
```

```
2
```

A short-cut is as follows

```
{a[[1]], b[[1]], b[[1, 1]]}
```

```
{2, {1, 2}, 1}
```

To form a list one can employ the "Table" command

```
Table[x + 2 y, {x, 5}]
```

```
{1 + 2 y, 2 + 2 y, 3 + 2 y, 4 + 2 y, 5 + 2 y}
```

```
Table[f[x], {x, 5}]
```

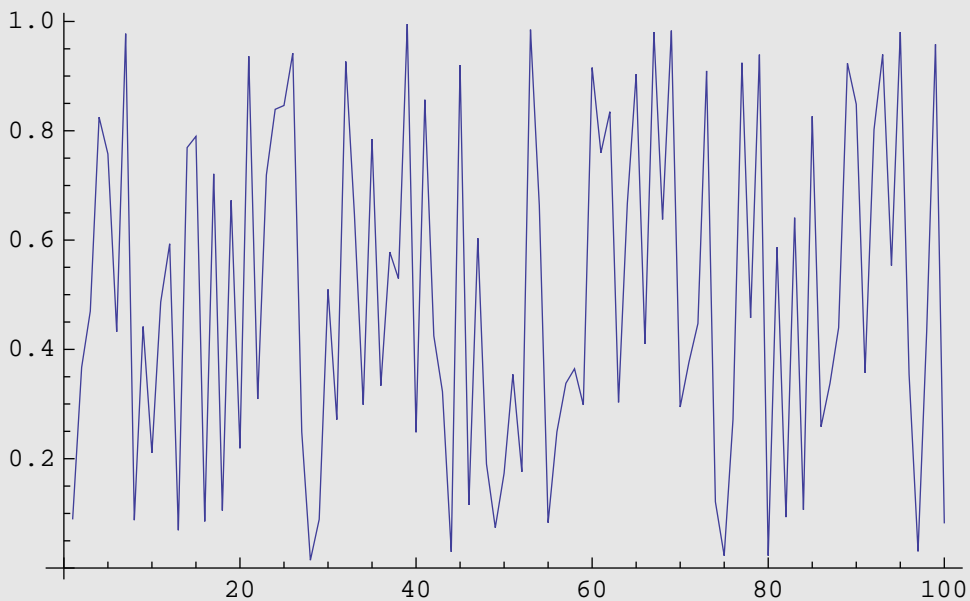
```
{f[1], f[2], f[3], f[4], f[5]}
```

Here we first produce a list of random numbers

```
iLength = 100; listRandom = Table[Random[], {i, 1, iLength}];
```

use the command "ListPlot" to visualize the result of this random sampling

```
ListPlot[listRandom, PlotJoined → True]
```



and then calculate the mean and the standard deviation of this sample



$$\text{sampleMean} = \frac{1}{\text{iLength}} \sum_{i=1}^{\text{iLength}} \text{listRandom}[[i]]$$

$$\text{sampleDeviation} = \left( \frac{1}{\text{iLength} - 1} \sum_{i=1}^{\text{iLength}} (\text{listRandom}[[i]] - \text{sampleMean})^2 \right)^{1/2}$$

```
0.501975
```

```
0.313544
```

The same can be done using the built-in functions "Mean" and "StandardDeviation"

```
{Mean[listRandom], StandardDeviation[listRandom]}
```

```
{0.501975, 0.313544}
```

## Functions

*Mathematica* contains a number of built-in-functions. In addition, one can easily define new functions. The general form of a function definition is  $f[x_] := \text{body}$ . Note that the function argument  $x$  is followed by an underscore. The combination  $x_$  is called a pattern. Also note the "special" definition symbol  $:=$ .

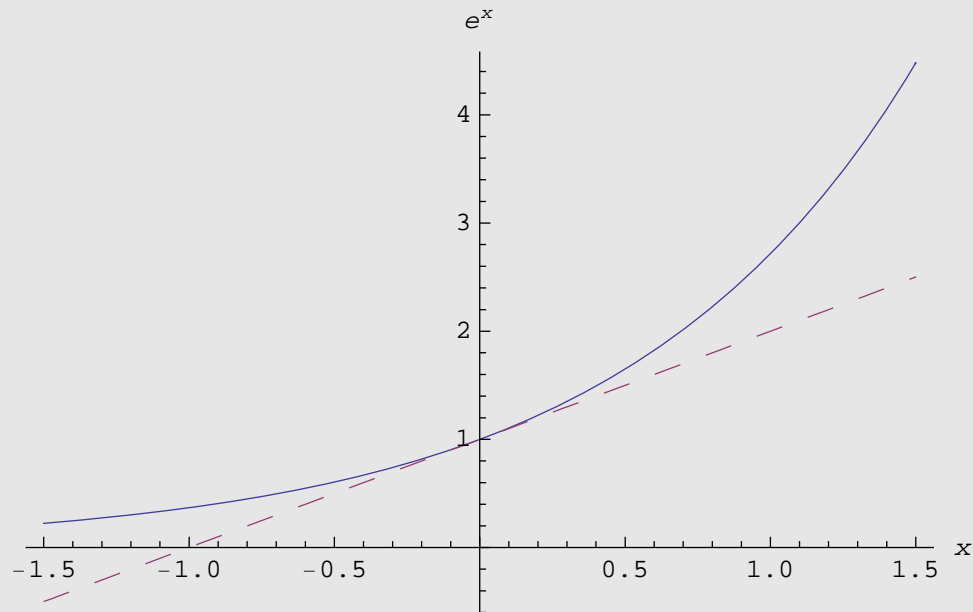
**Built-in functions.** Here is a plot of  $f(x) = e^x$  together with its first-order Taylor series approximation about  $x = 0$

```
series1 = Series[Exp[x], {x, 0, 1}]
series2 = Normal[series1]
```

```
1 + x + O[x]^2
```

```
1 + x
```

```
Plot[{Exp[x], series2}, {x, -1.5, 1.5}, AxesLabel -> {x, e^x},
PlotStyle -> {{}, Dashing[{0.03}]}
```



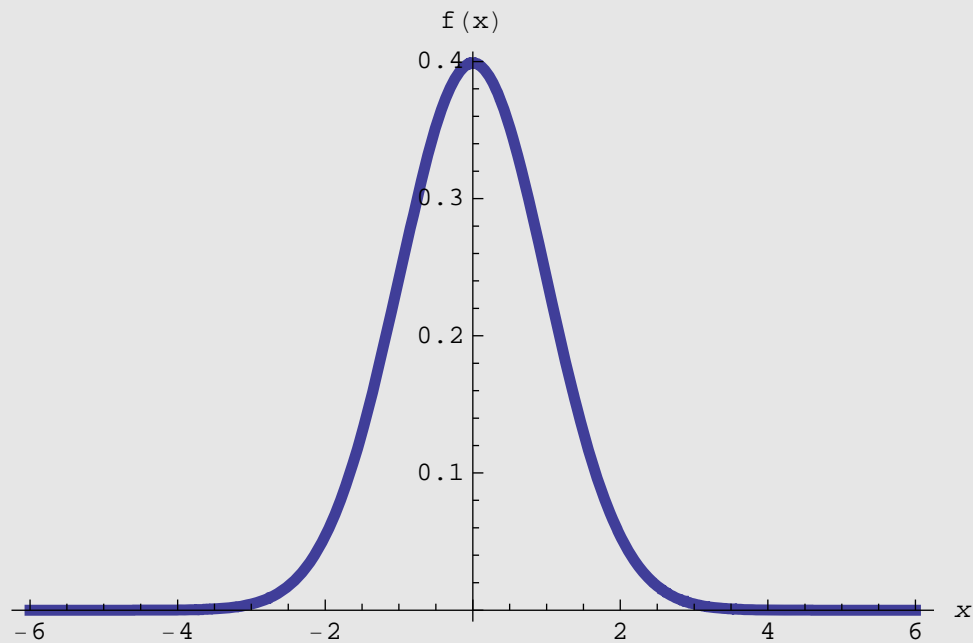
Notice that, in dynamic macroeconomics, we frequently use the approximation  $e^x \simeq 1 + x$ , implying  $x \simeq \ln(1 + x)$ , for "small values" of  $x$ .

**Newly-defined functions 1.** Here we define a Gaussian function with unity variance and zero mean

$$f[x_] := \frac{1}{\sqrt{2\pi}} \text{Exp}\left[-\frac{x^2}{2}\right]$$

To visualize, let us plot this function

```
Plot[f[x], {x, -6, 6}, PlotStyle -> Thickness[0.012],
  AxesLabel -> {x, "f(x)"}]
```



Our new function may now be used like any other *Mathematica* function.

```
 $\partial_x f[x]$ 
```

$$-\frac{e^{-\frac{x^2}{2}} x}{\sqrt{2\pi}}$$

$$\int_{-\infty}^{\infty} f[x] dx$$

```
1
```

The "probability mass" between  $x \in [-1.96, 1.96]$  is

$$\int_{-1.96}^{1.96} f[x] dx // N$$

```
0.950004
```

**Newly-defined functions 2.** Here we define a function that calculates the mean of a list of numbers.

$$\text{mean}[\mathbf{x}_] := \frac{1}{\text{Length}[\mathbf{x}]} \sum_{i=1}^{\text{Length}[\mathbf{x}]} \mathbf{x}[[i]]$$

Here we test the function

```
mean[Table[Random[], {100 000}]]
```

```
0.499891
```

## Some functional programming (available upon request)

### 5. Differential and difference equations

#### Differential equations

Here is a simple linear, autonomous differential equation (DE). Since it is linear, it can be solved analytically yielding a closed-form solution.

```
Clear[a, b]; dex = x'[t] == a + b x[t];
```

```
sol = DSolve[dex, x[t], t]
```

```
{ { x[t] -> -a/b + e^{b t} C[1] } }
```

Observe that *Mathematica* names the arbitrary constant of integration  $C[1]$ .

We visualize the result by using an example. First, we extract the solution from the above output expression

```
toplot1 = sol[[1, 1, 2]];
```

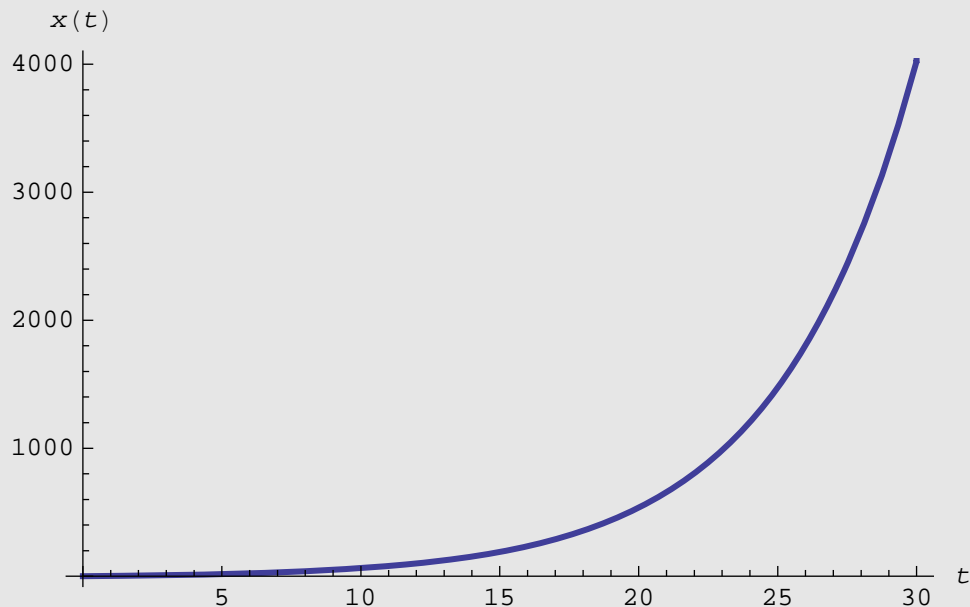
Here we replace the constant of integration ( $C[1]$ ) by the numerical value "10"

```
toplot = toplot1 /. C[1] -> 10
```

```
-a/b + 10 e^{b t}
```

Specifying parameters  $a$  and  $b$  numerically, we can use `Plot` command to plot the result

```
Clear[a, b]; a = 2; b = 0.2;
Plot[toplot, {t, 0, 30}, AxesLabel -> {t, x[t]},
PlotStyle -> {Thickness[0.007]}]
```



*Reference information:* Abell and Braselton (2004) is an extremely helpful book on differential equations with *Mathematica*.

## Difference equations

Here is a simple difference equation, which can be solved analytically by using "RSolve"

```
Clear[a, b]; sol = RSolve[{x[t] == a x[t - 1] + b, x[1] == 1}, x[t], t]
```

$$\left\{ \left\{ x[t] \rightarrow \frac{-a^t + a^{1+t} - a b + a^t b}{(-1 + a) a} \right\} \right\}$$

To plot the solution we first construct a list of  $x[t]$ , evaluated at integer  $t$ -values, using "Table"

```
list = Table[sol[[1, 1, 2]], {t, 0, 20}]
```

$$\left\{ \frac{-1+a+b-ab}{(-1+a)a}, \frac{-a+a^2}{(-1+a)a}, \frac{-a^2+a^3-ab+a^2b}{(-1+a)a}, \right.$$

$$\frac{-a^3+a^4-ab+a^3b}{(-1+a)a}, \frac{-a^4+a^5-ab+a^4b}{(-1+a)a}, \frac{-a^5+a^6-ab+a^5b}{(-1+a)a},$$

$$\frac{-a^6+a^7-ab+a^6b}{(-1+a)a}, \frac{-a^7+a^8-ab+a^7b}{(-1+a)a}, \frac{-a^8+a^9-ab+a^8b}{(-1+a)a},$$

$$\frac{-a^9+a^{10}-ab+a^9b}{(-1+a)a}, \frac{-a^{10}+a^{11}-ab+a^{10}b}{(-1+a)a}, \frac{-a^{11}+a^{12}-ab+a^{11}b}{(-1+a)a},$$

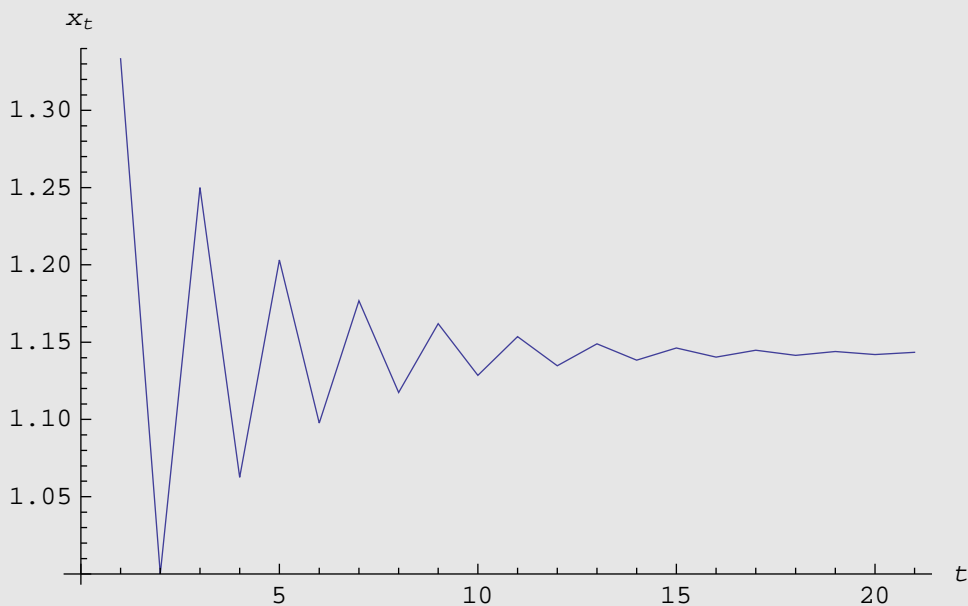
$$\frac{-a^{12}+a^{13}-ab+a^{12}b}{(-1+a)a}, \frac{-a^{13}+a^{14}-ab+a^{13}b}{(-1+a)a}, \frac{-a^{14}+a^{15}-ab+a^{14}b}{(-1+a)a},$$

$$\frac{-a^{15}+a^{16}-ab+a^{15}b}{(-1+a)a}, \frac{-a^{16}+a^{17}-ab+a^{16}b}{(-1+a)a}, \frac{-a^{17}+a^{18}-ab+a^{17}b}{(-1+a)a},$$

$$\left. \frac{-a^{18}+a^{19}-ab+a^{18}b}{(-1+a)a}, \frac{-a^{19}+a^{20}-ab+a^{19}b}{(-1+a)a}, \frac{-a^{20}+a^{21}-ab+a^{20}b}{(-1+a)a} \right\}$$

```
Clear[a, b]; a = -0.75; b = 2;
```

```
ListPlot[list, PlotJoined → True, AxesLabel → {t, x_t}]
```



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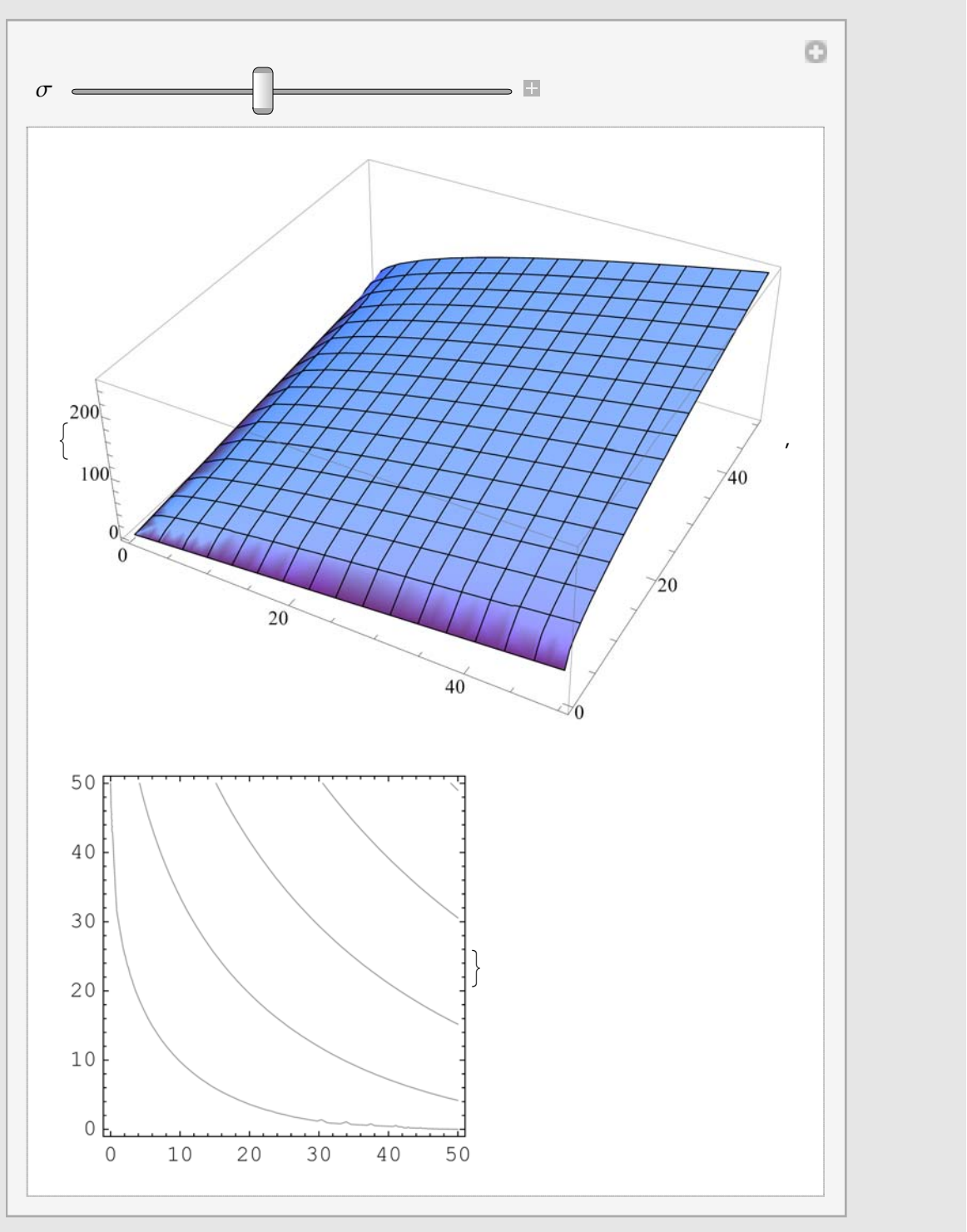
## 6. Graphics

### An isoquant plot of CES functions

Especially for teaching it might be helpful to plot isoquants of a CES technologies, like  $Y = (x_1^\sigma + x_2^\sigma)^{1/\sigma}$ . There are two basic possibilities to do this.

#### Possibility 1

```
Manipulate[ {Plot3D[ (x1σ + x2σ)1/σ, {x1, 0.001, 50}, {x2, 0.001, 50}],  
ContourPlot[ (x1σ + x2σ)1/σ, {x1, 0.001, 50}, {x2, 0.001, 50},  
ContourShading → False] }, {σ, -100, 1}]
```



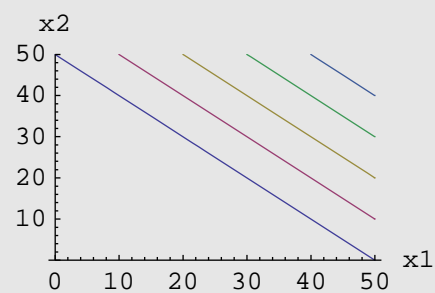
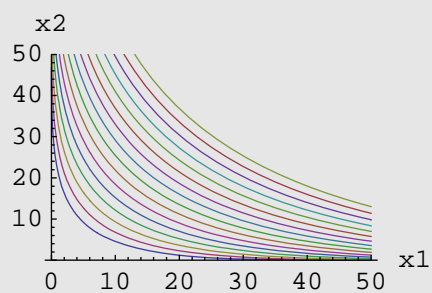
Possibility 2



```

Clear[α, p1, p2]; α = .35;
p1 =
Plot[Evaluate[Table[Solve[(x1α + x2α)1/α == Y, x2][[1, 1, 2]],
  {Y, 50, 200, 10}]], {x1, 0.001, 50}, PlotRange → {0, 50},
  AxesLabel → {x1, x2}];
Clear[α]; α = 1;
p2 =
Plot[Evaluate[Table[Solve[(x1α + x2α)1/α == Y, x2][[1, 1, 2]],
  {Y, 50, 200, 10}]], {x1, 0.001, 50}, PlotRange → {0, 50},
  AxesLabel → {x1, x2}];
Show[GraphicsArray[{p1, p2}]]

```



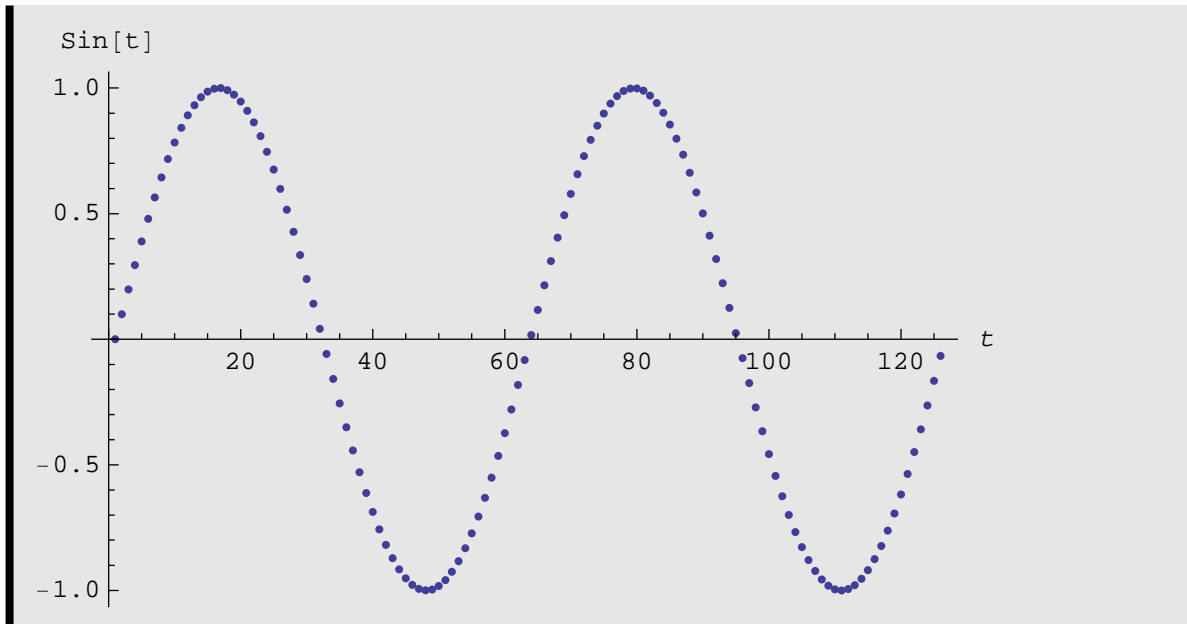
## Plotting data

Here is a list of data points

```
dataList = Table[Sin[t], {t, 0, 4 π, 0.1}];
```

This list can be plotted using `ListPlot`

```
ListPlot[dataList, AxesLabel → {t, "Sin[t]"}]
```



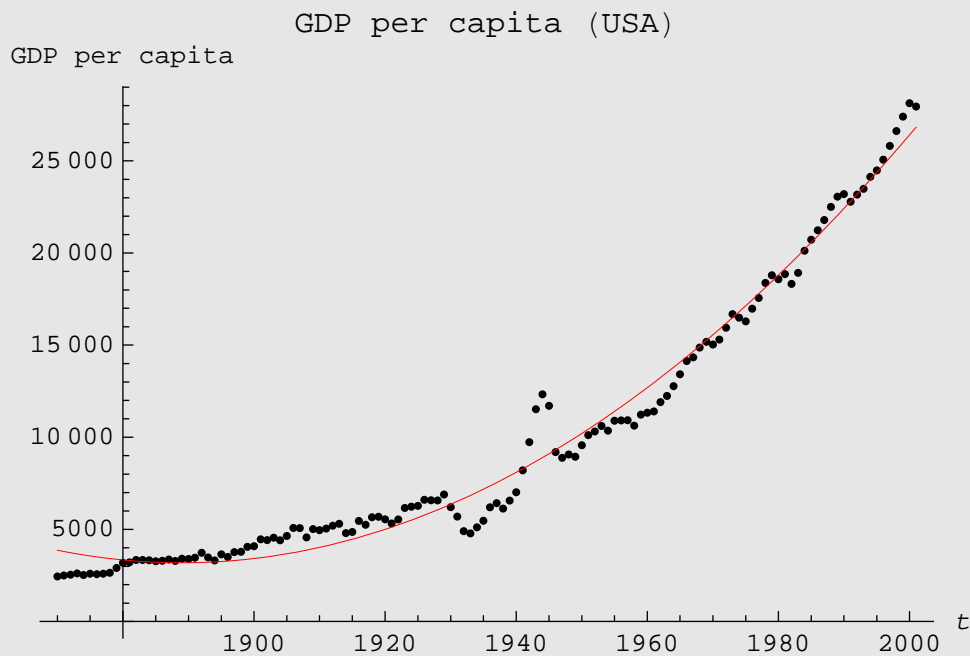
What about empirical data series? This can be done by employing `Import[url]`. To try this, we put an excel data file at:

[http://www.wifa.uni-leipzig.de/fileadmin/user\\_upload/itvwl-vwl/makro/Lehre/qdm/-GDP\\_Per\\_Capita\\_USA.xls](http://www.wifa.uni-leipzig.de/fileadmin/user_upload/itvwl-vwl/makro/Lehre/qdm/-GDP_Per_Capita_USA.xls)

```

list =
  Import [
    "http://www.wifa.uni-leipzig.de/fileadmin/user_upload/itvwl-vwl/
      makro/Lehre/qdm/GDP_Per_Capita_USA.xls"];
p1 = ListPlot[list, PlotStyle -> {{Thickness[0.03], Black}},
  AxesLabel -> {t, "GDP per capita"}];
Clear[a, b, c];
parameterFit = FindFit[list[[1, 1 ;; 132, 2]], a + b x + c x^2,
  {a, b, c}, x];
trend = Table[a + b x + c x^2 /. parameterFit, {x, 132}];
gdpcTrend = Thread[{list[[1, 1 ;; 132, 1]], trend}];
trendPlot = ListLinePlot[gdpcTrend,
  PlotStyle -> {Thickness[0.001], Red},
  AxesLabel -> {t, "GDP per capita"}];
Show[p1, trendPlot, PlotLabel -> "GDP per capita (USA)"]

```



## 7. Supplement: notation and basic calculation

Taken from Jankowski, 1998, Lecture 3 (<http://usm.maine.edu/~mjankowski/docs/ele298/index.htm>)



`f/@expr` is equivalent to `Map[f, expr]`

`stringa <> stringb` is equivalent to `StringJoin[stringa, stringb]`

## ■ Punctuation

As in C and other programming languages the semicolon (;) is a command terminating character. Additionally, it is used to block kernel output to screen. The comma (,) is used as a simple separator. *Mathematica* allows multiple statements in a cell or even a single line. The characters "(" and ")" denote beginning and end of comments.

## ■ Function names

*Mathematica* is case sensitive.

If you know what you want to do but you don't know the name of the *Mathematica* function to use, some rules can help you guess. (Of course, you could always look it up in the Help Browser, but who wants to read documentation when you can guess instead?)

- All function names start with capital letters, and multi-word names use internal capitalization (the way German should but doesn't). There is an exception for widely accepted abbreviations of common functions such as `Sin`, `Log`, `Exp`, etc.
- Nothing is ever abbr. (except N and D). `Integer` is `Integer` not `Int`. `Integrate` is `Integrate`, not `Int`.
- Functions named after a person are the person's name plus the commonly used symbol. Examples:

`BesselJ`

`LegendreP`

`ChebyshevT`

Exception:

`Zeta` (should have been `RiemannZ`)

- When in doubt, think long. Examples:

`FactorInteger`

`LinearProgramming`

`MathieuCharacteristicExponent`

A *Mathematica* variable or symbol may consist of any number of contiguous letters and numbers. The name should not start with a number. Some special characters should not be used: underscore character, ampersand, period. Here are a few examples of BAD variable names:

```
{x_y, x.y, x & y}
```

## ■ Constants

Built-in constants adhere to this convention: I, E, Pi, Infinity (or in the newer forms:  $i$ ,  $e$ ,  $\pi$ ,  $\infty$ ). The Traditional-Forms are all obtained via the Escape key.

```
I I
```

```
N[E]
```

```
N[π]
```

```
 $\frac{1}{\infty}$ 
```

```
0 ∞
```

## Basic symbolic and numeric calculations

We begin with a discussion of symbolic vs. numeric calculations based on material in Glynn/Gray, Chapters 5 and 23.

### ■ Simple calculations

Even at this early stage, you will encounter some interesting examples since most of you are familiar with numerical calculations only.

```
1 + π
```

```
N[%]
```

```
N[%, 50]
```

```
 $\frac{1}{3}$ 
```

```
N[%]
```

Note the difference between the following two results. You need to be aware of the fact, that when using real numbers or **N**, all calculations are approximate (within the precision of computers floating-point representation).

```
Sin[ $\frac{\pi}{3}$ ]
```

```
Sin[ $\frac{\pi}{3.}$ ]
```

The following example will give another example of the difference between *Mathematica's* exact and approximate number representations.

```
M = { { { 11, 1 } / 16, { 1, 11 } / 2 }, { 1/3, -11/5 } };
M.Inverse[M]
```

```
N[M].Inverse[N[M]]
```

Some more symbolics using complex numbers.

```
Sqrt[-4]
```

```
Exp[I Pi / 2]
```

```
{ Re[Exp[I Pi / 3]], Im[Exp[I Pi / 3]] }
```

```
Conjugate[Exp[I Pi / 3]]
```

```
ExpToTrig[Exp[I Pi / 3]]
```

The value of a variable need not be a number. Here is another example.

```
y = 2 x + 1
```

```
y^2 + 2 x
```

Here is a method of checking your basic math skills. The == symbol is a relational operator, it tests for equality.

```
Log[10, Pi] == Log[Pi] / Log[10]
```

```
Cos[x]^2 + Sin[x]^2 == 1
```

The last example shown what happens when *Mathematica* does NOT understand the input - it simply returns the input unevaluated (see below for the proper way of testing this well known trigonometric identity).

```
TrigToExp[Cos[x]^2 + Sin[x]^2] == 1
```

Other relational operators are: >, <, >=, <=.

## ■ Algebra and calculus

Two result in the preceding section did not yield the desired results. Here we will try again.

```
Expand[y^2] - 4 x
```

```
Simplify[Cos[x]^2 + Sin[x]^2] == 1
```

Expand and Simplify are two examples of a large family of functions useful in manipulating algebraic expressions.

We move on to examples from calculus. Note that you can place comments on *Mathematica* input lines.

```
D[x^n] (* this is a derivative *)
```

```
D[{x, 2} x^n]
```

```
Integrate[% dx] (* % stands for the last output *)
```

Here is a "favorite" of electrical engineering students. It gives the inverse Fourier transform of the ideal lipase filter of bandwidth  $1 \frac{\text{rad}}{\text{sec}}$ .

$$\text{sinc} = \frac{1}{2\pi} \int_{-1}^1 \text{Exp}[I \omega t] d\omega$$

Follow with some algebraic manipulation and we get

```
ExpToTrig[%]
```

There are however many integrals for which no explicit formula can be given. These can be solved numerically.

$$\int \sin[\sin[3x]] dx$$

Definite integrals have the same general form. However, the second argument is now an iterator. It has the general form {x, xmin, xmax}.

```
NIntegrate[Sin[Sin[3x]], {x, 0, 1}]
```